Fitness Landscape
And The Performance Of Local Search Algorithms

Philippe Preux Denis Robilliard
Cyril Fonlupt
Laboratoire d'Informatique du Littoral, BP 719, 62228 Calais Cedex
fonlupt@lil.univ-littoral.fr

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Though probably the most well-known $\mathcal{NP}$-hard problem and in spite of the experimental knowledge having been gathered for years, the structure of the research space associated to the Traveling Salesman Problem is still poorly known. For their part, local search algorithms (LSA) are also still far from being well understood. Theoretical results tell us that performance of LSA on the TSP can not be bounded [CK77]. However, it is an experimental evidence that a steepest descent hill-climber (SDHC) iteratively picking-up the best neighbor at each step (the neighborhood of a point being defined with regards to the 2-opt move, or k-opt move) performs much better than Christofides' algorithm with known bound of $1/2$. Neither simple questions with regards to the number of local optima, the distribution of their length and their location in the research space have yet received pragmatic answers – even though anything can happen in theory – nor has there been any attempt to relate this knowledge to the performance of various LSA on this problem. Our work aims at providing insights on these questions.

The notion of research space is closely related to the concept of fitness landscapes introduced by Sewall Wright in biology in 1932 [Wri32]. This concept has recently received attention to study adaptation embedded in adaptive algorithms [Kau93]. We have argued elsewhere (see [DPT96] for example) that each adaptive algorithms (LSA) is associated to a class of landscapes on which it "feels at ease".

We have performed statistics on a large number of hill-climbing walks on a set of instances of the ETSP taken from the TSPLib [Rei91] where we know a global optimum. They have shown us that the local optima (LO) are all gathered in the same region of the research space, thus forming a massif central in Kaufmann's terminology around the global optimum. Thus, starting out with a random tour, the hill-climber is driven towards the massif central and gets stuck on a LO of very good quality. We have compared the SDHC with other hill-climbers which selects one neighbor among the better points and have obtained better LO with SDHC Thus, the simplest gives the best results. Furthermore, the number of steps performed by the SDHC scales linearly with the size of the instance. The very good performance of the SDHC is in accord with the fact that the landscape is very simple. This landscape is also in accord with the fact that sophisticated hill-climbers such as evolutionary algorithms are very weak on these instances.

We think that asymetrical TSP and TSP based on non fully connected graphs will show other structures of landscape and that this structure will explain the performance of the different hill-climbers (among other things, it will contribute to a better understanding of recombination operators used in EA's) and give us some clues to derive better search strategies.

References

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